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ESTIMATING THE SAMPLING VARIANCE OF CORRELATION CORRECTED FOR ATTENUATION USING COEFFICIENT ALPHA

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Summary.—A formula derived by Kelley estimates the sampling variance of the correlation corrected for attenuation by using split-half reliabilities. In some cases, the coefficient alpha estimate of reliability will be preferred for use over the split-half. An examination of a recent Monte Carlo simulation suggests a variation of the Kelley formula can be used appropriately with coefficient alpha. Kelley's formula is modified to accept coefficient alpha directly.

This paper presents a formula for estimating the sampling variance of the correlation corrected for attenuation, for use when calculations utilize coefficient alpha as an estimate of reliability. The correction for attenuation, $r_{xy}/\sqrt{r_{xx}r_{yy}}$, gives an estimate of what the correlation between two random variables, X and Y, would be if the measure of each variable were perfectly reliable. The correction can be used to assess how high a correlation might rise if the experimenter increased the reliability of the measures involved. It can also be used to determine whether two tests measure different attributes (Cureton, 1965), which is the case when the corrected correlation is statistically significantly less than 1.0.

Calculating an exact estimate of the sampling variance for the correlation corrected for attenuation is regarded as "exceedingly difficult, if not impossible" (Rogers, 1976). Rogers recently demonstrated that Tukey's jackknife procedure (Tukey, 1958) can be applied to the problem, but this involves a moderate amount of computation time.

Existing formulae for approximating the variance of the corrected correlation have proven reasonably accurate under Monte Carlo simulations. However, they all use the split-half method of reliability. The method involves dividing the items of each test in half, correlating the halves, and correcting the correlation in order to get an estimate of the reliability of the full length test. The split-half method can generate as many estimates of reliability for a given test as there are ways to divide the test in half; the method also assumes equal variance of the test halves. Coefficient alpha (Cronbach, 1951), the generalized form of Kuder-Richardson formula 20 (Kuder & Richardson, 1937), has the advantage over the split-half method that there is one single reliability estimate for a given test, equal to the average of all possible split-half correlations; the assumptions underlying alpha are also less restrictive.

There are two major problems in utilizing coefficient alpha in variance formulae which accept split-half correlations. The first problem is one of test

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length. A correlation between two halves of a test represents the reliability of half the length of the test. Coefficient alpha, on the other hand, represents the reliability of the full-length test. Alpha must be adjusted downward to be comparable to a correlation between halves; this is done through a modification of one of the formulae below.

A second problem is one of sampling variance. Since the split-half reliability estimate is different from coefficient alpha in that it varies according to the different ways to split a test, and its underlying assumptions are different, its sampling distribution may also be somewhat different from that of coefficient alpha. The variance equations derived for use with split-half reliabilities can at best yield further approximations of the variance when used with coefficient alpha. However, there is good reason to believe the sampling distributions of the split-half and coefficient alpha estimates are quite similar, when assumptions are met. Both reliability estimates primarily reflect the intercorrelations among test items. Cronbach (1951, pp. 305-306) demonstrated that randomly chosen split-halves and coefficient alpha will tend to converge as the number of test items increases and concluded that "α measures essentially the same thing as the split-half coefficient." The foregoing suggests that interchanging the two reliability estimates in the variance formulae, once there is an appropriate correction for length, will not affect accuracy of the formulae to any meaningful extent. In many circumstances where alpha is contemplated for use, any loss in accuracy will be more than offset by the use of a more accurate estimate of reliability.

Forsyth and Feldt (1969) present a Monte Carlo simulation which tested different equations for estimating the sampling variance of the corrected correlation, given varying reliabilities and intercorrelations. In the simulation, there were two hypothetical tests, each composed of two items with a zero mean and unit variance. The only way to split each of the tests is with one item in each half, and therefore the model under which the formulae were tested included neither the variance due to different ways to split a test nor that due to unequal item variance. Both coefficient alpha and split-half methods yield the same full scale reliability estimates under such conditions.

Forsyth and Feldt concluded that the best equation for estimating the variance of the corrected correlation was one by Kelley (1947, p. 528). The Kelley equation reads:

$$V_{att} = [r_{att}^2/4(N-2)]\,[4r_{att}^2 + (4/r_{xy}^2) + (1/r_{ab}^2) + (1/r_{cd}^2) - (4/r_{ab}) - (4/r_{cd}) - 2] \quad [1]$$

where X and Y are random variables measured over population P, and A, B, C, and D are the respective scores on the two halves of each measure, and where lower-case letters are used for subscripting. Pearson product-moment correlations are represented by r_{xy}, r_{ab}, etc. The correlation corrected for attenuation is r_{att}, and its variance estimate is V_{att}. There are N observations. To modify the equation so as to accept coefficient alpha estimates directly, the Spearman-Brown formula is first solved for the correlation between the halves, yielding $r_{xx}/(2 - r_{xx})$ where r_{xx} is the reliability of test X. The expressions $r_{xx}/(2 - r_{xx})$ and $r_{yy}/(2 - r_{yy})$ can then be substituted into Equation [1], yielding, after simplification:

$$V_{att} = [r_{att}^2/(N-2)]\,[r_{att}^2 + (1/r_{xy}^2) + (1/r_{xx}^2) + (1/r_{yy}^2) - (3/r_{xx}) - (3/r_{yy}) + 2] \qquad [2]$$

Forsyth and Feldt suggest the standard error (SE) of the corrected correlation is best approximated by taking the square root of the above and multiplying by the correction $\sqrt{N/(N-2)}$. As an example, consider the case where two tests are administered to a sample ($N = 102$), and intercorrelate .86, with alpha reliabilities of .90 and .93 each. The correlation corrected for attenuation is .94. Formula [2] works out to be .000595. The SE would be the square root of this multiplied by the correction factor or .024635. Because the sampling distribution of corrected correlations is approximately normal when N is sufficiently large (Forsyth & Feldt, 1969), the 95% confidence interval around the correlation corrected for attenuation would extend from .892 to .988.

Hypothesis testing with the formula can be done utilizing the general method proposed by Forsyth and Feldt which involves substituting the hypothesized values of the corrected correlation ($H\rho_{att}$) for r_{att}, and substituting the hypothesized intercorrelation between the two tests (Hr_{xy}) for r_{xy} where:

$$Hr_{xy}^2 = H\rho_{att}^2\,(r_{xx}\,r_{yy})$$

but the alpha level may be only approximately met. For testing the difference of r_{att} from 1.0, the hypothesized disattenuated correlation would be 1.0. Following the example presented above where $r_{xx} = .90$ and $r_{yy} = .93$, $Hr_{xy} = .915$ and $V_{att} = .000264$, with $SE = .0164$.

In conclusion, the modified Equation [2] can serve as an approximate estimator of the variance with its characteristics noted in the Forsyth and Feldt simulation study, when coefficient alpha is to be used as an estimate of reliability.

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