

Pampered Pig Playland Technical Report
How to please all pigs with two kinds of slides

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[Note that signatures should also be here.]

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[Note to readers: The main purpose of this sample technical report is to show how similar reports should be written. The physics and mathematics here is relatively simple (but not trivial) so you can pay attention the form of the report and the pieces. All the comments in italics are to comment on the form of the report and are not part of the report itself.]

Introduction

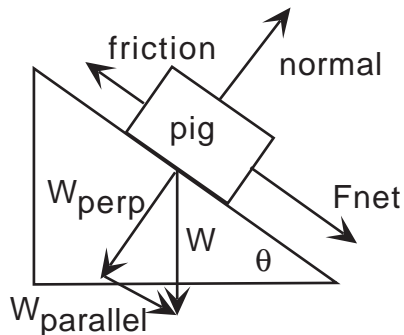
[The purpose of the introduction is to give a brief overview of the situation and the results. It should be just enough to prepare a first time reader for what is to come; all details should be in the following sections.]

The market research division of Pampered Pig Playland, a chain of playgrounds for pampered pigs, has found that some pigs are nearly fearless and enjoy the quickest ride possible down a slide. This fastest time can be achieved by greasing the slide, resulting in a coefficient of friction of $\mu = .05$. However, there are other pigs that are more timid and prefer to take twice the fastest time. The design division of P³ has found that, for the slides constructed with an angle of incline between 30 and 40 degrees, a coefficient of friction of μ between .45 and .64 will result in a slide that provides an enjoyable ride to the more hesitant patrons.

Physical Situation

[In this section we include a complete description of the situation, examine carefully all qualitative aspects of the system, include helpful sketches and plots. The purpose is to give a conceptual overview of the situation and the answer without any mathematical details. This will help a reader make sense of the next section.]

We approximate the slide as an inclined plane, and we can consider the pig as a block sliding down the plane. Below is the free body diagram that shows all of the forces acting on the pig: gravity, friction, and normal. We will ignore air resistance for the purposes of this report because we assume the speed of the pig remains quite small and so the force due to air resistance is small. We will check this assumption at the end of our work. *[Notice how we carefully state our assumptions.]*



[Almost all reports are enhanced by a visualization of some sort. Think carefully about what pictures, plots, sketches would be most useful to your readers.]

We know that the normal force exactly balances the component of the weight that is perpendicular to the slide so there is no acceleration perpendicular to the slide. *[Note that we are not explaining many things, such as forces and vector addition. You should assume that your reader has taken the same course you are taking and you don't need to explain what is in the book.]*

The net force is the component of the weight down the slide minus the frictional force since these forces are in opposite directions. The frictional force is equal to the coefficient of friction, μ , times the normal force.

To get some idea of the coefficient of friction for the slower slide, consider the following. The larger the frictional force, the smaller the net force, and the smaller the acceleration, since acceleration is equal to the net force divided by the mass. So the μ that we need for the slow slide must be larger than $\mu = .05$ of the fast slide. *[Here we have made some approximation of the answer, even before we calculate. Here it is trivial, but in other cases it probably won't be so.]*

The important parameter values are the mass of the pig m_{pig} , the length of the slide, l , and the acceleration due to gravity, g , and the angle θ of the incline. We will show later on that our results, fortunately, do not depend on the mass of the pig or the length of the slide.

We have two different coefficients of friction which we will denote by μ_{slow} and μ_{fast} , and two times of descent down the slide t_{slow} and t_{fast} . Our goal is to find μ_{slow} in terms of the other parameters of the problem. *[Note how we have used subscripts to differentiate similar variables. Also, we have restated the problem.]*

Mathematical Model

[In this section we state and solve the problem in mathematical terms. Details of the calculation are given. You should give enough detail so that another student could solve a similar problem. All assumptions should be clearly stated and justified. All of your conclusions should be supported.]

There are three main parts to the solution. First we use the constant acceleration equations to find a relation between acceleration and time. Then we use our knowledge of forces to relate μ to the acceleration. Then we put the last two results together, along with the constraint that the fast time be half the slow time to get a relation between μ_{slow} and the other parameters of the problem. This approach will work because it allows us to relate the unknown μ_{slow} to the known parameter values using basic relations between position, velocity, acceleration and time.

[It is often useful to give a roadmap to the reader for this section, since this section can get quite long and involved. This is also a good place to justify the approach you just outlined.]

We begin by connecting the time of travel to the distance traveled and to the acceleration of the pig. We know this is possible because acceleration is the second derivative of position with respect to time.

This is a case of constant acceleration, because the forces involved are constant. The forces would not be constant if the slide were curved (then the angle of incline constantly changes), but with a constant incline, the acceleration is constant. *[Note how we carefully justify the use of the constant acceleration equations, and comment on when they would not be applicable.]*

Therefore we may use the constant acceleration equations:

$$x(t) = x(0) + v(0)t + \frac{1}{2}at^2$$

[We have typed the equations into our text. You can either learn how to type equations – it's not that hard! – or write them by hand. The later may be the only real choice if you have pages and pages of equations. Also, if you don't know how to insert pictures and plots directly in your text, that's okay. Just be sure they are easy for the reader to find.]

We take $x(0)$, the initial position of the pig, to be zero. This is allowed because we can put the origin of our coordinate system anywhere. We also will assume that $v(0)$, the initial velocity of the pig, is also zero, as this is typically the case. In any case, it is sure to be the case for the timid pigs. Also $x(t)$ is the final position of the pig, which is the length of the slide l in both cases. With all this in mind, we now have the following equations:

$$l = \frac{1}{2}a_{fast}t_{fast}^2$$

$$l = \frac{1}{2}a_{slow}t_{slow}^2$$

$$t_{slow} = 2 t_{fast}$$

where the last line is from our initial constraint on the times. Equating the first two lines, and plugging in the last line we get

$$a_{fast}t_{fast}^2 = a_{slow}(2 t_{fast})^2$$

$$a_{fast} = 4a_{slow}$$

giving a relation between the fast and slow accelerations.

We now turn to finding how the accelerations depend on the angle and coefficients of friction. The frictional force is μ times the normal force. The normal force equals the

perpendicular component of the weight, which is $mg\cos\theta$. As a check, note that if the incline is actually flat, so that $\theta=0$, then the normal force equals the weight, which is correct. So we have

$$F_{friction} = \mu mg \cos\theta$$

The component of the weight parallel to the slide is given by

$$W_{parallel} = mg \sin\theta$$

As a check, if $\theta=90$ degrees, all of the weight is parallel to the incline.

The net force is given by the vector sum of the individual forces. Since the friction and parallel component of the weight are in opposite directions, we subtract to get the net force and acceleration

$$F_{net} = mg \sin\theta - \mu mg \cos\theta$$

$$F_{net} = mg(\sin\theta - \mu \cos\theta)$$

$$a = \frac{F_{net}}{m} = g(\sin\theta - \mu \cos\theta)$$

We see now that the mass of the pig does not effect the acceleration.

Now we combine this expression for the individual accelerations with the relation given above between the accelerations:

$$a_{fast} = 4a_{slow}$$

$$g(\sin\theta - \mu_{fast} \cos\theta) = 4g(\sin\theta - \mu_{slow} \cos\theta)$$

$$(\sin\theta - \mu_{fast} \cos\theta) = 4(\sin\theta - \mu_{slow} \cos\theta)$$

$$\sin\theta - \mu_{fast} \cos\theta = 4\sin\theta - 4\mu_{slow} \cos\theta$$

$$4\mu_{slow} \cos\theta = 3\sin\theta + \mu_{fast} \cos\theta$$

$$\mu_{slow} = \frac{3 \tan\theta + \mu_{fast}}{4}$$

Here we have our value of μ_{slow} in terms of μ_{fast} and the angle. Note that g , l , and m_{pig} are irrelevant. This is quite fortunate, as we do not want to have to ask pigs their weight before they get on the slide and adjust μ_{slow} for each pig.

Missing: a plot of μ_{slow} as a function of θ .

The plot show how μ_{slow} changes as a function of θ . Note that as θ increases, so does μ_{slow} .

[Plots are often a good way of summarizing data. Note how the plots axes are labeled with units and the plot is given a title.]

Checks

[In this section we give the reader as many reasons as we can to believe that our answer is correct. This is an essential part of the report. Imagine millions of dollars rest on these results!]

Of course we need to check our answer to ensure that the calculations are correct so that we do not cause our patrons any emotional or physical discomfort.

First, we note that as μ_{fast} increases, so does μ_{slow} . This makes sense because as μ_{fast} increases, the time for a pig to go down the fast slide increases, and so time to go down the slow slide must also increase.

Second, we note that as θ gets closer to 90 degrees, tangent θ increases and μ_{slow} increases. This makes sense because the normal force is getting weaker as the slide becomes more vertical, therefore μ_{slow} must increase to compensate.

Third, we note that the units on μ_{slow} are correct – it is unitless as required.

Finally, we will actually plug in some numbers to see that the times are in the desired relation. We will take $l=1$ m, $\theta=35$ degrees, calculate μ_{slow} and then both t_{fast} and t_{slow}

$$\mu_{slow} = \frac{3 \tan \theta + \mu_{fast}}{4}$$

$$\mu_{slow} = \frac{3 \tan 35 + .05}{4}$$

$$\mu_{slow} = .536$$

Now we will check the times for both μ_{slow} and μ_{fast} :

$$l = \frac{1}{2} a t^2$$

$$t = \sqrt{\frac{2l}{a}}$$

$$a = g(\sin \theta - \mu \cos \theta)$$

$$t = \sqrt{\frac{2l}{g(\sin \theta - \mu \cos \theta)}}$$

$$t_{fast} = \sqrt{\frac{2 * 1m}{9.8m/s^2(\sin 35 - .05 \cos 35)}}$$

$$t_{fast} = .619s$$

$$t_{slow} = \sqrt{\frac{2 * 1m}{9.8m/s^2(\sin 35 - .536 \cos 35)}}$$

$$t_{slow} = 1.24s$$

And so we see that indeed that t_{fast} is half of t_{slow} .

Lastly, we need to check our initial assumption that air resistance (also known as the drag force) is small. We find that the drag force is given by the following equation [1]: *[We have put a footnote here to show you how to reference materials. You should give a reference to any information that is not part of the textbooks we are using. We do not have a particular bibliographic style in mind – just be sure to give enough information that a fellow student could find the information on their own.]*

$$F_{drag} = \frac{1}{2} C \rho A v^2$$

We take $C=.7$, since the pig is not particularly aerodynamic in shape (C is usually between 0 and one, with smaller values for more aerodynamic shapes). We take A = cross sectional area of the pig to be $.09 \text{ m}^2$. ρ is the density of air, and is known to be 1.21 kg/m^3 . V is the velocity of the pig. Since this is always changing, we take it to be the maximum velocity so that we obtain an upper bound on the size of the drag force. (Recall that the initial velocity is zero.) We take numbers from our previous check.

$$v(t) = at$$

$$v(1.24s) = 9.8m/s^2 * (\sin 35 - .536 * \cos 35) * 1.24s$$

$$v(1.24s) = 1.63m/s$$

$$F_{drag} = \frac{1}{2} C \rho A v^2$$

$$F_{drag} = \frac{1}{2} * .7 * 1.21kg/m^3 * .09m^2 * (1.63m/s)^2$$

$$F_{drag} = .1kg \cdot m/s^2 = .1N$$

This is quite small compared with typical weights (50 pounds is about 200 N, so even at an angle of 35 degrees, the parallel component of the weight is 114N). In any event, the drag force will slow the pig down, so at worst the fearless pigs are less satisfied by their ride, but the timid pigs are still not terrified.

Conclusion

[The conclusion conveys much of the same information as the introduction, but in slightly more technical terms, now that the reader is more knowledgeable about the problem.. This is also the place to suggest possible refinements and open questions.]

Pampered Pig Playland has two slides in each of its parks nationwide. We plan to devote one slide to the fearless pigs. This slide will be greased to provide an exceptionally fast ride, with coefficient of friction $\mu=.05$.. The more timid pigs will be able to use a slide that takes twice as long to go down as the fast slide. This can be achieved if the coefficient of friction of the slow slide is as follows:

$$\mu_{slow} = \frac{3 \tan \theta + \mu_{fast}}{4}$$

where θ is the angle of incline of the slide.

The next step in this project is for the materials department to find materials that will achieve the proper coefficient of friction with your average pig.

Bibliography

1. D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics, Sixth Edition*, (John Wiley and Sons, New York, 2001), p 104.